Quiz 4
EECS 203
Spring 2015

Name (Print): _____

uniqname (Print): _____

Instructions. You have 25 minutes to complete this quiz. You may not use any sources of information, including electronic devices, textbooks, or notes. Leave at least one seat between yourself and other students. Please write clearly. If we cannot read your writing, it will not be graded.

Honor Code. This course operates under the rules of the College of Engineering Honor Code. Your signature endorses the pledge below. After you finish your exam, please sign on the line below:

I have neither given nor received aid on this examination, nor have I concealed any violations of the Honor Code.

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Page #	Points
1	/18
2	/12
Total	**/30**

1) Circle all of the following that are **TRUE**. **(4 points, -2 per incorrectly circled/non-circle, minimum 0)**

- <mark>Perhaps the largest difference between a public and private key system is that a private key system relies upon having shared a "key" securely ahead of time, while a public key scheme does not.</mark>

- The RSA scheme covered in class makes it difficult to convert between the public key and the private key because to do so requires finding the prime factorization of the encryption exponent "e".

- <mark>In RSA, the public key, "n" is generally the product of two very large primes.</mark>

- It is very common to use private key encryption to exchange public keys.

2) How many bit strings of length 7 either start with "111" or end with "11" (or both)? You are to give your answer as a number and show your work. Partial credit for an incorrect answer will be rare. **[4]**

There are 2^4 strings with 111 at the start (choose 0 or 1 for each of the other 4 digits) and 2^5 strings with 11 at the start (choose 0 or 1 for each of the other 5 digits).

We can count the number of bit strings satisfying either of these by adding the number of strings satisfying each and subtracting strings satisfying both (as these will be double-counted). There are 2^2 strings satisfying both (choose 0 or 1 for the 4th and 5th digit).

In total, the answer is 16+32-4 = 44

3) Use Fermat's Little Theorem to find 7^{123} mod 11. **[4]**

FLT: 7^{10} == 1 mod 11
So 7^{123} == (7^10)^12 (7^3) == 1^12 (7^3) == 49*7 == 5*7 == 2 mod 11

4) How many poker hands consist of 2 pairs? That is two of one rank of card, two of another rank of card, and one of a third rank of card? Recall that a poker hand consists of 5 cards and a deck of cards consists of 4 suits each with one card of each of the 13 ranks. You should leave your answer as an equation. **[6]**

(13 choose 2)(4 choose 2)^2*44

We choose two ranks to have pairs in, then choose the two suits for each pair (the first 4 choose 2 for the smaller rank chosen, the second for the larger rank), then choose among the 44 cards not of the chosen 2 ranks to be the 5th card.

5) Prove or disprove the following statement:

A computer network consists of five computers. Each computer is directly connected to at least one of the other computers. Show that there are at least two computers in the network that are directly connected to the same number of other computers. **[8 points]**

There are 5 computers, each of which have either 1, 2, 3, or 4 connections (each computer has at most 4 other computers to connect to). By the pigeonhole principle, there must be some number between 1 and 4 such that 2 or more computers have this number of connections.

6) How many bit strings of length 7 have a substring of "111" (so at least 3 ones in a row) in it? (Note: this question is hard). Your answer should be a number and you should have a clear explanation as to how you got that number. Partial credit for an incorrect answer will be rare. **[4]**

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Two ways to get this:

There are 5 strings with 3 1s that have 111 (just choose a starting location for 3 1s)
There are 5*4-4 = 16 strings with 4 1s (choose a starting location for 3 1s, choose location for "the other 1", then subtract out the 4 strings with 4 consecutive 1s)
There are (7 choose 5) – 3 = 18 strings with 5 1s (only strings with 5 1s but no 3-runs are 1011011, 1101011, 1101101)
There are (7 choose 6) = 7 strings with 6 1s, and (7 choose 7) = 1 string with 7 1s, all of which have a run of 3 1s.
Total 5+16+18+7+1 = 47.

OR

There are 16 strings of form 111XXXX,
8 strings of the form 0111XXX,
8 strings of the form X0111XX,
8 strings of the form XX0111X,
and 8 strings of the form XXX0111, but we need to subtract out 1110111 since it's counted twice here.
Total 16+8+8+8+8-1 = 47